Federal Univesity of Technology – Paraná (UTFPR)

A New Predictor-Corrector Method for Optimal Power Flow

Roy Wilhelm Probst

27-28 April, 2015 - Campinas, Brazil

Affine-Scaling and Central Path Methods

$$\begin{array}{ll} \min & c^t x \\ \text{s.t.} & Ax = b \\ & x \geq 0 \end{array}$$

$$\begin{bmatrix} A & 0 & 0 \\ 0 & A^t & I \\ Z & 0 & X \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} = \begin{bmatrix} b - Ax \\ c - A^t z \\ -XZu \end{bmatrix}$$

or

$$\begin{bmatrix} A & 0 & 0 \\ 0 & A^t & I \\ Z & 0 & X \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} = \begin{bmatrix} b - Ax \\ c - A^t z \\ \mu u - XZu \end{bmatrix}$$

Affine-Scaling and Central Path Methods

$$\begin{aligned} \min \quad & c^t x \\ \text{s.t.} \quad & Ax = b \\ & x \geq 0 \end{aligned}$$

$$\begin{bmatrix} A & 0 & 0 \\ 0 & A^t & I \\ Z & 0 & X \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} = \begin{bmatrix} b - Ax \\ c - A^t z \\ -XZu \end{bmatrix}$$

or

$$\begin{bmatrix} A & 0 & 0 \\ 0 & A^t & I \\ Z & 0 & X \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} = \begin{bmatrix} b - Ax \\ c - A^t z \\ \mu u - XZu \end{bmatrix}$$

Predictor-Corrector Method

$$
\begin{array}{ll}
\min & c^t x \\
\text{s.t.} & Ax = b \\
 & x \geq 0
\end{array}
$$

$$
\begin{bmatrix} A & 0 & 0 \\ 0 & A^t & I \\ Z & 0 & X \end{bmatrix}
\begin{bmatrix} \Delta\tilde{x} \\ \Delta\tilde{y} \\ \Delta\tilde{z} \end{bmatrix}
=
\begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix}
$$

$$
\begin{bmatrix} A & 0 & 0 \\ 0 & A^t & I \\ Z & 0 & X \end{bmatrix}
\begin{bmatrix} \Delta\hat{x} \\ \Delta\hat{y} \\ \Delta\hat{z} \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ \mu u - \Delta\tilde{X}\Delta\tilde{Z}u \end{bmatrix}
$$

PC Method for Nonlinear Programming

$$\begin{array}{ll} \min & f(x) \\ \text{s.t.} & g(x) = 0 \\ & x \geq 0 \end{array}$$

$$\begin{bmatrix} \nabla_x g(x) & 0 & 0 \\ \nabla^2_{xx} L(x,y,z) & \nabla_x g(x)^t & I \\ Z & 0 & X \end{bmatrix} \begin{bmatrix} \Delta\tilde{x} \\ \Delta\tilde{y} \\ \Delta\tilde{z} \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix}$$

$$\begin{bmatrix} \nabla_x g(x) & 0 & 0 \\ \nabla^2_{xx} L(x,y,z) & \nabla_x g(x)^t & I \\ Z & 0 & X \end{bmatrix} \begin{bmatrix} \Delta\tilde{x} \\ \Delta\tilde{y} \\ \Delta\tilde{z} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \mu u - \Delta\tilde{X}\Delta\tilde{Z}u \end{bmatrix}$$

PC Method for Nonlinear Programming

$$\begin{array}{rc} \min & f(x) \\ \text{s.t.} & g(x) = 0 \\ & x \geq 0 \end{array}$$

$$\begin{bmatrix} \nabla_x g(x) & 0 & 0 \\ \nabla^2_{xx} L(x,y,z) & \nabla_x g(x)^t & I \\ Z & 0 & X \end{bmatrix} \begin{bmatrix} \Delta\tilde{x} \\ \Delta\tilde{y} \\ \Delta\tilde{z} \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix}$$

$$\begin{bmatrix} \nabla_x g(x) & 0 & 0 \\ \nabla^2_{xx} L(x,y,z) & \nabla_x g(x)^t & I \\ Z & 0 & X \end{bmatrix} \begin{bmatrix} \Delta\tilde{x} \\ \Delta\tilde{y} \\ \Delta\tilde{z} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \mu u - \Delta\tilde{X}\Delta\tilde{Z}u \end{bmatrix}$$

Complete PC – Corrections

$$\begin{aligned} \min \quad & x^t H x \\ \text{s.t.} \quad & XAx = b \\ & x \geq 0 \end{aligned}$$

$$\begin{aligned} XZu &= 0 \\ Z\Delta x + X\Delta z &= -XZu \\ (X + \Delta X)(Z + \Delta Z)u &= \Delta X \Delta Z u \end{aligned}$$

Complete PC – Corrections

$$\begin{aligned} \min \quad & x^t H x \\ \text{s.t.} \quad & XAx = b \\ & x \geq 0 \end{aligned}$$

$$\begin{aligned} XZu &= 0 \\ Z\Delta x + X\Delta z &= -XZu \\ (X + \Delta X)(Z + \Delta Z)u &= \Delta X \Delta Z u \end{aligned}$$

Primal Corrections

$$\begin{aligned} \min \quad & x^t H x \\ \text{s.t.} \quad & XAx = b \\ & x \geq 0 \end{aligned}$$

$$\begin{aligned} XAx - b &= 0 \\ (XA + diag(Ax))\Delta x &= b - XAx \\ (X + \Delta X)A(x + \Delta x) - b &= \Delta X A \Delta x \end{aligned}$$

Dual Corrections

$$\begin{aligned} \min \quad & x^t H x \\ \text{s.t.} \quad & XAx = b \\ & x \geq 0 \end{aligned}$$

$$\begin{aligned} 2Hx + A^t Yx + YAx &= 0 \\ (2H + A^t Y + YA)\Delta x + (A^t X + diag(Ax))\Delta y &= -2Hx - A^t Yx - YAx \\ \cdots &= A^t \Delta Y \Delta x + \Delta YA\Delta x \end{aligned}$$

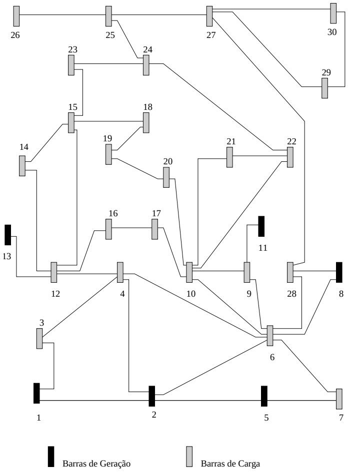
Complete Predictor-Corrector Method

$$\begin{aligned} \min \quad & x^t H x \\ \text{s.t.} \quad & XAx = b \\ & x \geq 0 \end{aligned}$$

$$\begin{bmatrix} XA + diag(Ax) & 0 & 0 \\ 2H + YA + A^t Y & A^t X + diag(Ax) & I \\ Z & 0 & X \end{bmatrix} \begin{bmatrix} \Delta\tilde{x} \\ \Delta\tilde{y} \\ \Delta\tilde{z} \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix}$$

$$\begin{bmatrix} \cdots & 0 & 0 \\ \cdots & \cdots & I \\ Z & 0 & X \end{bmatrix} \begin{bmatrix} \Delta\hat{x} \\ \Delta\hat{y} \\ \Delta\hat{z} \end{bmatrix} = \begin{bmatrix} -\Delta\tilde{X} A \Delta\tilde{X} u \\ -\Delta\tilde{Y} A \Delta\tilde{X} u - A^t \Delta\tilde{Y} \Delta\tilde{X} u \\ \mu u - \Delta\tilde{X} \Delta\tilde{Z} u \end{bmatrix}$$

IEEE 30 bus system



$$T_k = e_k + jf_k$$

- e_k : Voltage Real part
- f_k : Voltage Complex part
- P_k : Active Power liquid injection
- Q_k : Reactive Power liquid injection

Model

$$
\begin{array}{lll}
\min & e^t Ge + f^t Gf & \\
\text{s.t.} & P_k(e,f) + P_{C_k} - P_{G_k} = 0 & \forall k \in C \\
& Q_k(e,f) + Q_{C_k} - Q_{G_k} = 0 & \forall k \in C \\
& (v_k^{\min})^2 \leq V_k(e,f) \leq (v_k^{\max})^2 & \forall k \in N \\
& P_k^{\min} \leq P_k(e,f) \leq P_k^{\max} & \forall k \in G \\
& Q_k^{\min} \leq Q_k(e,f) \leq Q_k^{\max} & \forall k \in R
\end{array}
$$

where:

$$
\begin{array}{rcl}
P & = & EGe + FGf + FBe - EBf \\
Q & = & FGe - EGf - EBe - FBf \\
V & = & Ee + Ff
\end{array}
$$

CPC Corrections

$$\hat{r}_1 = \begin{bmatrix} \nabla_x P(\Delta\tilde{x})^t \Delta\tilde{y}_p + \nabla_x P(\Delta\tilde{x})^t \Delta\tilde{z}_{1_p} - \nabla_x P(\Delta\tilde{x})^t \tilde{z}_{2_p} \\ \nabla_x Q(\Delta\tilde{x})^t \Delta\tilde{y}_q + \nabla_x Q(\Delta\tilde{x})^t \Delta\tilde{z}_{1_q} - \nabla_x Q(\Delta\tilde{x})^t \tilde{z}_{2_q} \end{bmatrix}$$

$$\hat{r}_2 = \begin{bmatrix} \Delta\tilde{S}_{1_v}\Delta\tilde{z}_{1_v} - \mu u \\ \Delta\tilde{S}_{1_p}\Delta\tilde{z}_{1_p} - \mu u \\ \Delta\tilde{S}_{1_q}\Delta\tilde{z}_{1_q} - \mu u \end{bmatrix} \quad \hat{r}_3 = \begin{bmatrix} \Delta\tilde{S}_{2_v}\Delta\tilde{z}_{2_v} - \mu u \\ \Delta\tilde{S}_{2_p}\Delta\tilde{z}_{2_p} - \mu u \\ \Delta\tilde{S}_{2_q}\Delta\tilde{z}_{2_q} - \mu u \end{bmatrix}$$

$$\hat{r}_4 = \begin{bmatrix} V(\Delta\tilde{x}) \\ P(\Delta\tilde{x}) \\ Q(\Delta\tilde{x}) \end{bmatrix} \quad \hat{r}_5 = \begin{bmatrix} V(\Delta\tilde{x}) \\ P(\Delta\tilde{x}) \\ Q(\Delta\tilde{x}) \end{bmatrix} \quad \hat{r}_6 = \begin{bmatrix} -V(\Delta\tilde{x}) \\ -P(\Delta\tilde{x}) \\ -Q(\Delta\tilde{x}) \end{bmatrix}$$

CPC Corrections (cont.)

where:

$$
\begin{aligned}
P(\Delta\tilde{x}) &= \Delta\tilde{E}G\Delta\tilde{e} + \Delta\tilde{F}G\Delta\tilde{f} + \Delta\tilde{F}B\Delta\tilde{e} - \Delta\tilde{E}B\Delta\tilde{f} \\
Q(\Delta\tilde{x}) &= \Delta\tilde{F}G\Delta\tilde{e} - \Delta\tilde{E}G\Delta\tilde{f} - \Delta\tilde{E}B\Delta\tilde{e} - \Delta\tilde{F}B\Delta\tilde{f} \\
V(\Delta\tilde{x}) &= \Delta\tilde{E}\Delta\tilde{e} + \Delta\tilde{F}\Delta\tilde{f} \\
\nabla_x P(\Delta\tilde{x}) &= \begin{bmatrix} \Delta\tilde{E}G + \Delta\tilde{F}B + diag(G\Delta\tilde{e} - B\Delta\tilde{f}) \\ \Delta\tilde{F}G - \Delta\tilde{E}B + diag(G\Delta\tilde{f} + B\Delta\tilde{e}) \end{bmatrix} \\
\nabla_x Q(\Delta\tilde{x}) &= \begin{bmatrix} \Delta\tilde{F}G - \Delta\tilde{E}B - diag(G\Delta\tilde{f} + B\Delta\tilde{e}) \\ -\Delta\tilde{E}G - \Delta\tilde{F}B + diag(G\Delta\tilde{e} - B\Delta\tilde{f}) \end{bmatrix}
\end{aligned}
$$

OPF using Cartesian coordinates

- Quadratic objective function
- Quadratic constraints
- Constant Hessian
- Nonlinear corrections in all optimality conditions

Computational Experiments

- MATLAB 7.8 (R2009a)
- Windows XP
- Intel Core 2 Quad Q9550 2,83 GHz
- 3,23 GB of RAM

Power systems test cases

	Buses and lines				
System	$\|N\|$	$\|A\|$	$\|R\|$	$\|C\|$	$\|M\|$
IEEE14	14	5	5	9	20
IEEE30	30	6	6	24	41
IEEE118	118	54	54	64	186
BRAZIL	2257	201	201	2056	3509

Iteration count and CPU time

$v_k \in [0.90, 1.10]$

	Iterations			CPU Time		
System	PD	PC	CPC	PD	PC	CPC
IEEE14	11	11	8	0.03	0.03	0.02
IEEE30	12	9	8	0.04	0.03	0.03
IEEE118	17	17	15	0.44	0.46	0.41

Iteration count and CPU time

$v_k \in [0.95, 1.05]$

	Iterations			CPU Time		
System	PD	PC	CPC	PD	PC	CPC
IEEE14	12	10	9	0.03	0.02	0.02
IEEE30	12	9	9	0.04	0.03	0.03
IEEE118	23	24	18	0.60	0.65	0.49

Active constraints v_k at optimal solution

System	$v_k \in [0.90, 1.10]$		$v_k \in [0.95, 1.05]$	
	$v^{\max}$	$v^{\min}$	$v^{\max}$	$v^{\min}$
IEEE14	0	0	0	0
IEEE30	1	0	1	0
IEEE118	0	0	1	10

Iteration count and CPU time

BRAZIL system

$\sum P^{max} / \sum P_C$	Iterations PD	Iterations PC	Iterations CPC	CPU Time PD	CPU Time PC	CPU Time CPC
1.10	26	26	17	80.61	83.30	55.46
1.15	24	28	16	74.39	90.68	52.41
1.20	20	35	18	62.59	113.59	58.99

Conclusions

- Quadratic objective function and constraints
  - OPF Problem
- Complete Predictor-Corrector
  - Nonlinear Corrections
- Robustness and Speed
  - Heuristic